Starter

A circle with centre C has equation $(x+3)^2+(y-2)^2=25$.

(a) Write down:

(i) the coordinates of C; *(2 marks)*

(ii) the radius of the circle. *(1 mark)*

(b) (i) Verify that the point $N(0, -2)$ lies on the circle. *(1 mark)*

(ii) Sketch the circle. *(2 marks)*

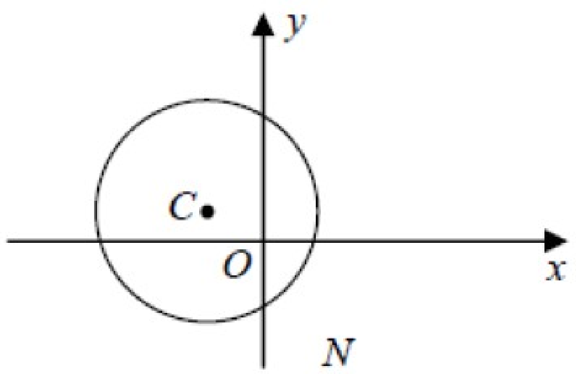
(iii) Find an equation of the normal to the circle at the point N. *(3 marks)*

(c) The point P has coordinates $(2, 6)$.

(i) Find the distance PC, leaving your answer in surd form. *(2 marks)*

(ii) Find the length of a tangent drawn from P to the circle. *(3 marks)*

Starter

5(a)(i)	Centre $(-3, 2)$	M1 A1	 2	± 3 or ± 2 correct
(ii)	Radius $= 5$	B1	1	accept $\sqrt{25}$ but not $\pm\sqrt{25}$
(b)(i)	$3^2 + (-4)^2 = 9 + 16 = 25$ $\Rightarrow N$ lies on circle	B1	1	must have $9 + 16 = 25$ or a statement
(ii)		M1 A1	 2	must draw axes; ft their centre in correct quadrant correct (reasonable freehand circle enclosing origin)

Starter

(iii)	Attempt at gradient of CN	M1		withhold if subsequently finds tangent
	grad $CN = -\frac{4}{3}$	A1		CSO
	$y = -\frac{4}{3}x - 2$ (or equivalent)	A1√	3	ft their grad CN
(c)(i)	$P(2,6)$ Hence $PC^2 = 5^2 + 4^2$	M1		"their" PC^2

	$\Rightarrow PC = \sqrt{41}$	A1	2	
(ii)	Use of Pythagoras correctly	M1		
	$PT^2 = PC^2 - r^2 = 41 - 25$, where T is a point of contact of tangent	A1√		ft their PC^2 and r^2
	$\Rightarrow PT = 4$	A1	3	**Alternative** sketch with vertical tangent M1 showing that tangent touches circle at point (2, 2) A1 hence $PT = 4$ A1

E Trigonometry

E3

Understand and use the sine, cosine and tangent functions; their graphs, symmetries and periodicity.

Know and use exact values of sin and cos for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ and multiples thereof, and exact values of tan for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \pi$ and multiples thereof.

Teaching guidance

Students should be able to:

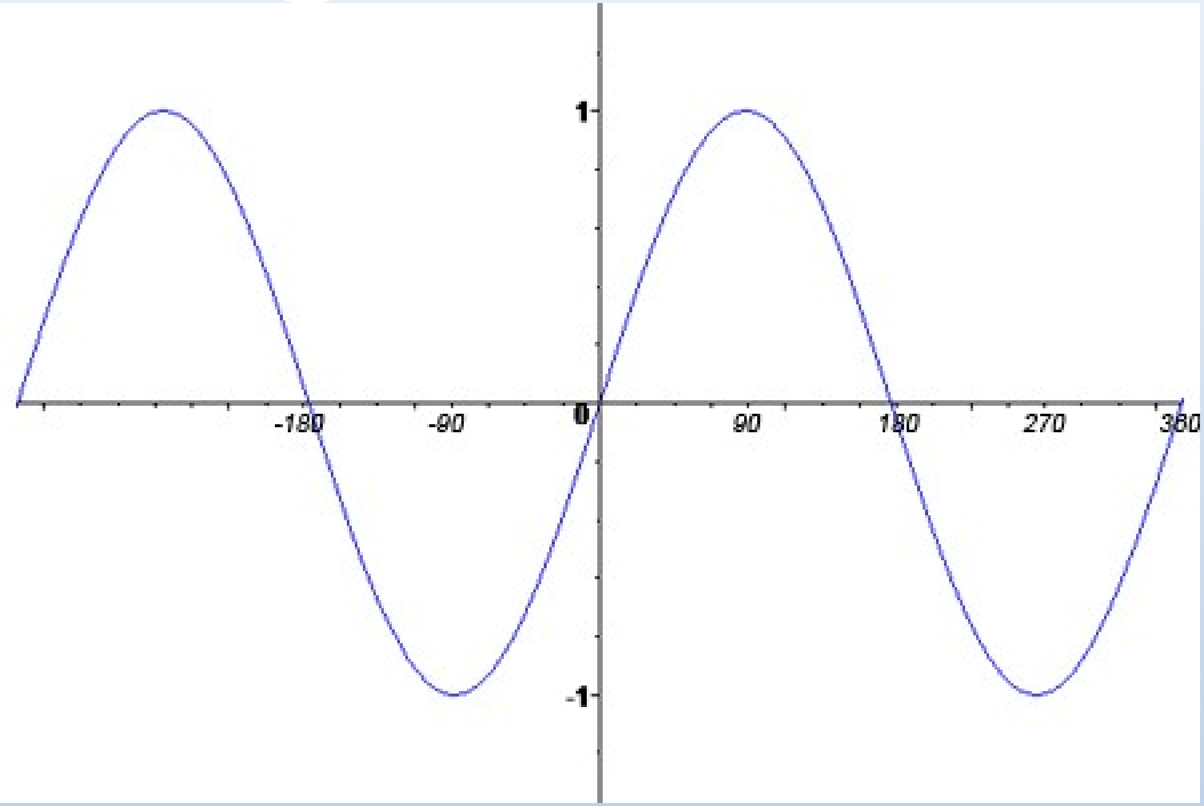
- understand and use vertical asymptotes of a tangent graph
- carry out simple transformations (as given in section B9) of the graphs of the sine, cosine and tangent functions.

At A-level combinations of transformations may be used.

Note: radians will not be required at AS.

3.1 Sine, cosine and tangent

The Sine Graph



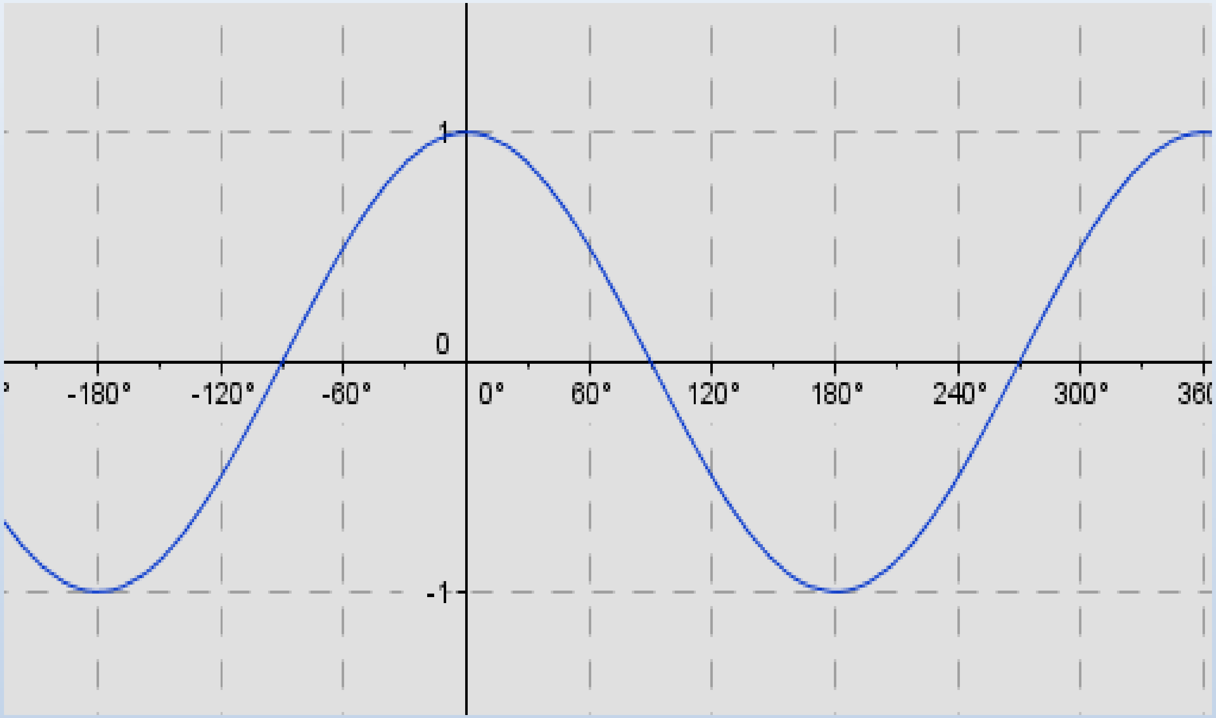
A periodic function with a period of 360 degrees

Lines of symmetry at:

Rotational symmetry (order 2) about every point where the graph intersects the -axis

3.1 Sine, cosine and tangent

The Cosine Graph



A periodic function with a period of 360 degrees

Lines of symmetry at:

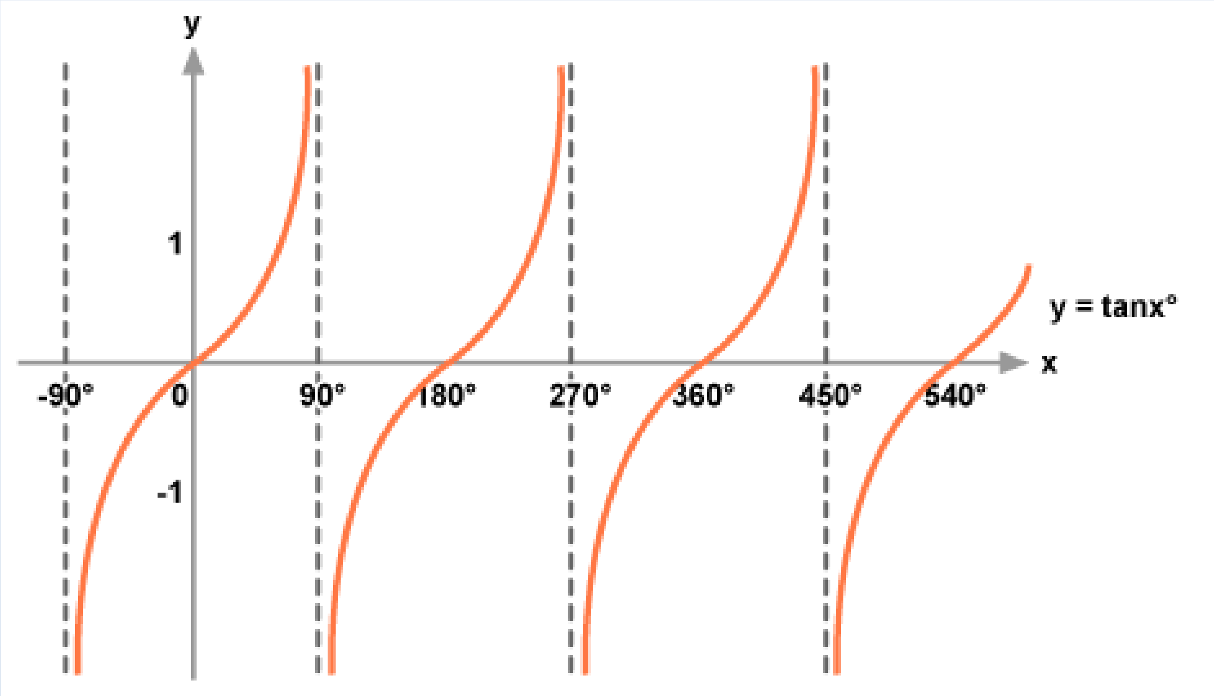
Rotational symmetry (order 2) about every point where the graph intersects the -axis

3.1 Sine, cosine and tangent

The Tangent Graph

A periodic function with a period of 180 degrees

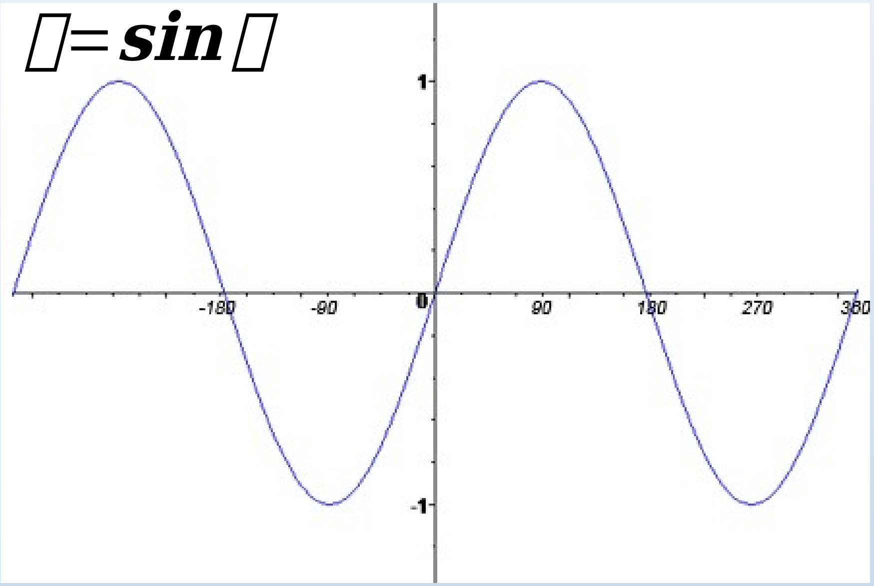
Asymptotes at



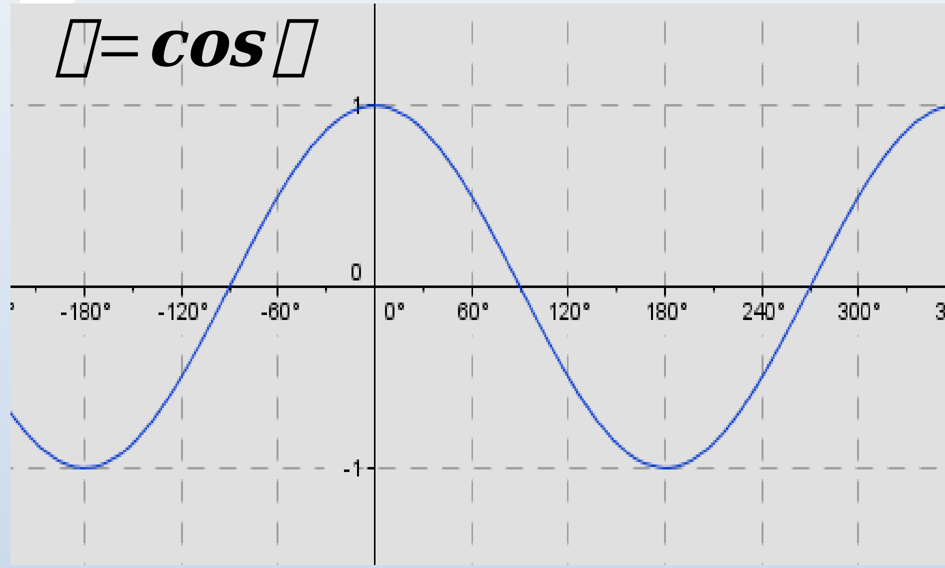
Rotational symmetry (order 2) about every point where the graph intersects the -axis

3.1 Sine, cosine and tangent

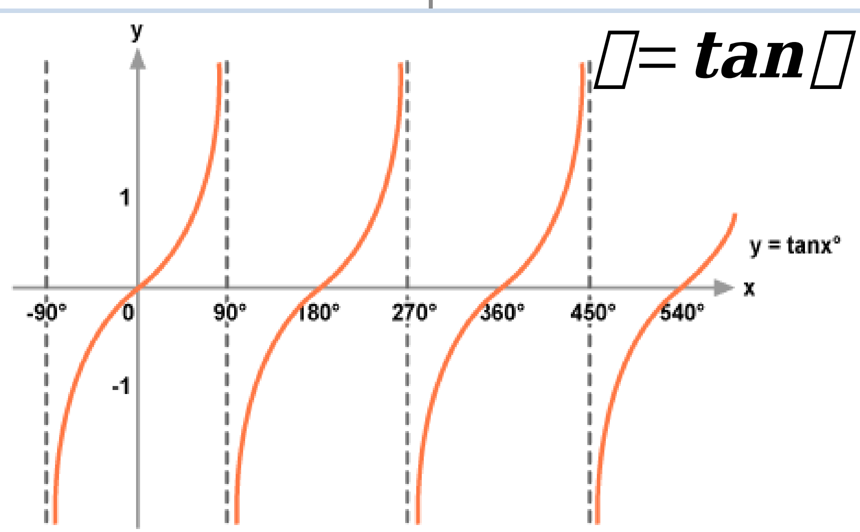
□= sin□

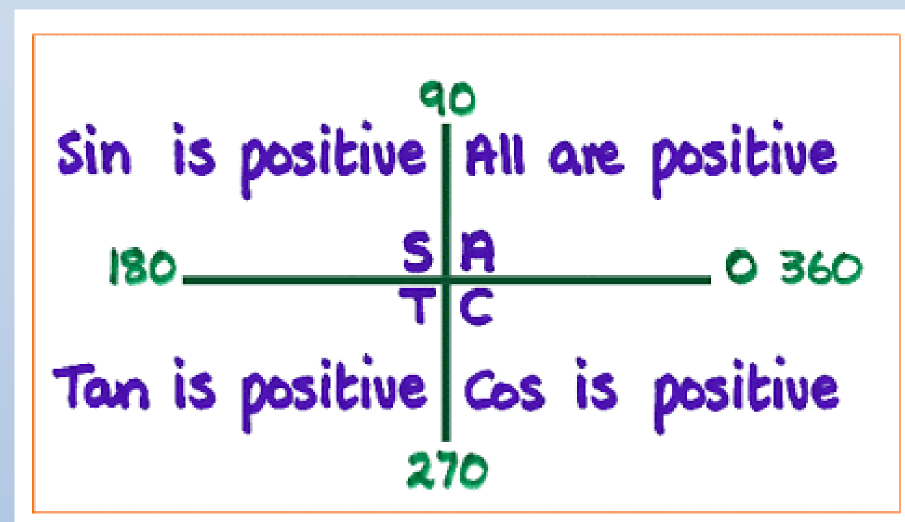


□= cos□



□= tan□



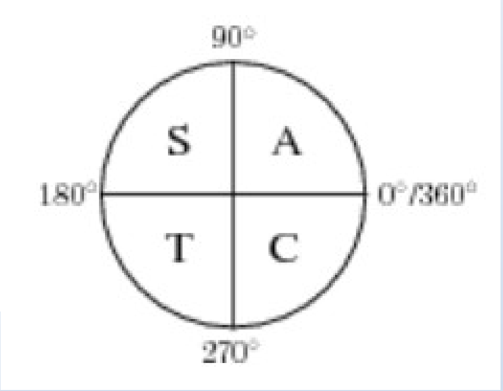


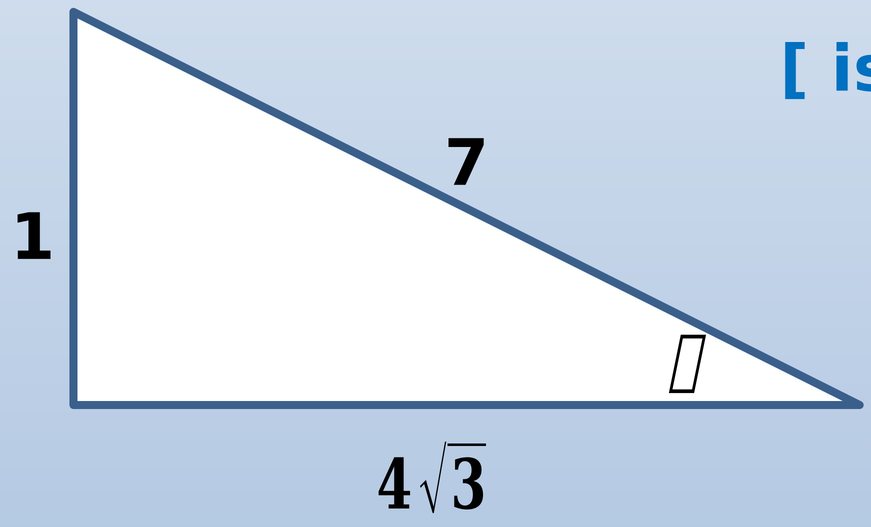
3.1 Sine, cosine and tangent

Example 1

Angle θ is such that $\sin\theta = \dfrac{1}{7}$ and $0 < \theta < 90°$

Find the exact value of $\tan\theta$





[is acute so tan is positive]

$$\therefore \tan\theta = \frac{1}{4\sqrt{3}} = \frac{\sqrt{3}}{12}$$

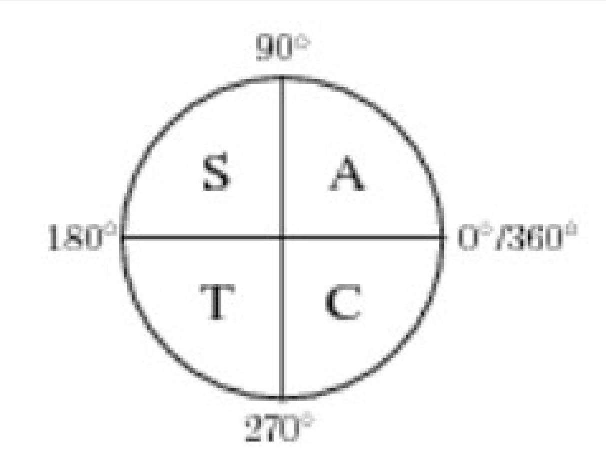
3.1 Sine, cosine and tangent

Example 2

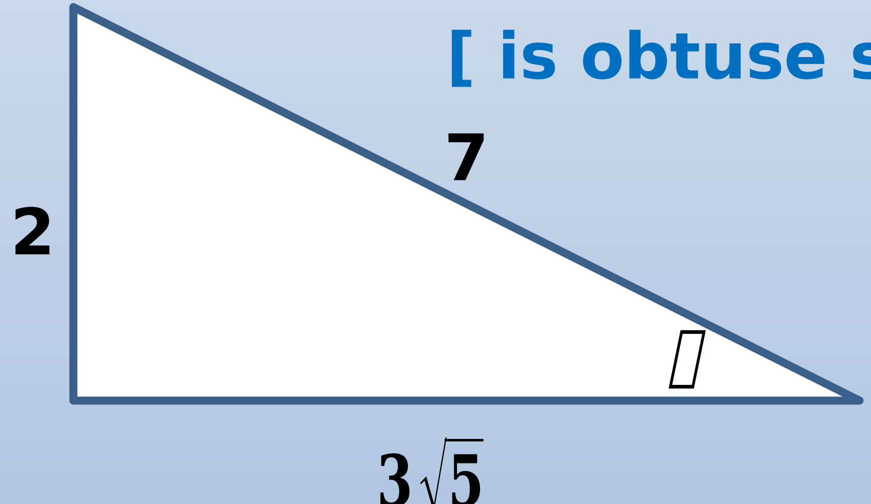
Given that $\sin\theta = \frac{2}{7}$ and θ is obtuse, find:

(a) the exact value of $\cos\theta$

(b) the exact value of $\tan\theta$



[is obtuse so only sin is positive]



3.1 Sine, cosine and tangent

Transforming Trig Graphs

Trigonometric graphs can be transformed in the same way as other graphs...

e.g. $y = \sin(\theta) + 1$ → translation of $\sin(\theta)$ by vector

$y = \sin(\theta + 30)$ → translation of $\sin(\theta)$ by vector

$y = 2\sin(\theta)$ → stretch in y-direction of $\sin(\theta)$ sf 2

3.1 Sine, cosine and tangent

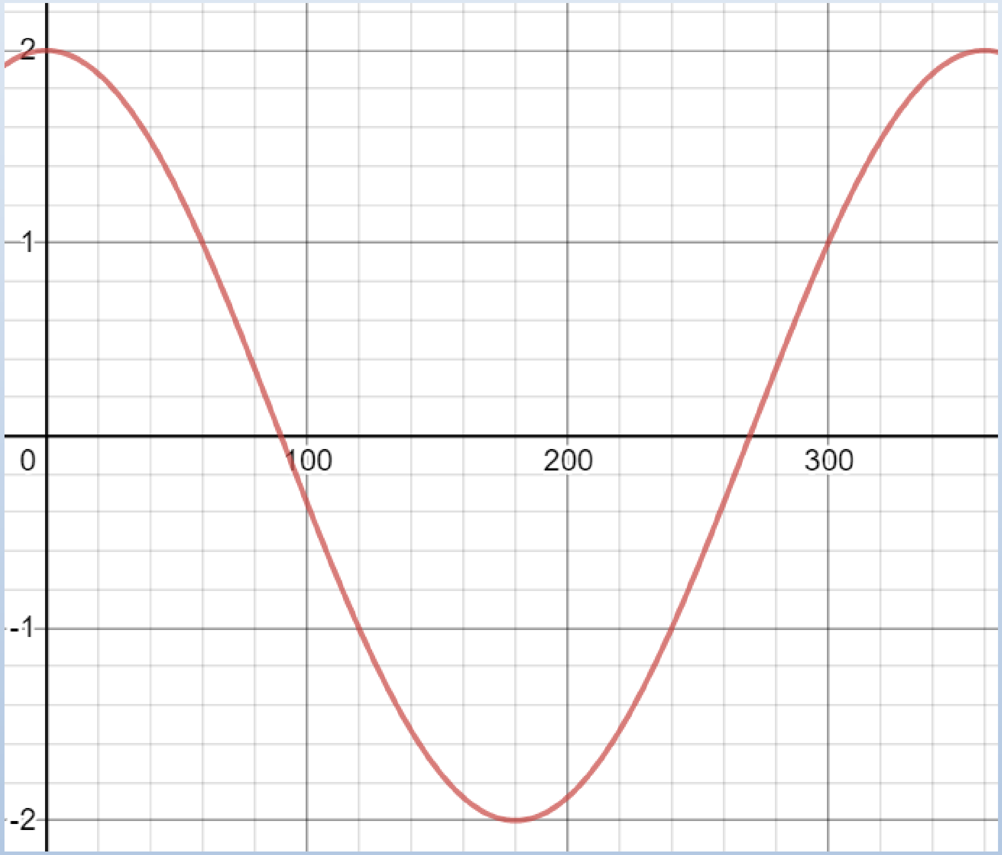
Example 3a

Sketch the graphs of the following from :

a)

b)

c)



Graph of stretched parallel to the -axis with scale factor 2.

i.e. all the -coordinates double

3.1 Sine, cosine and tangent

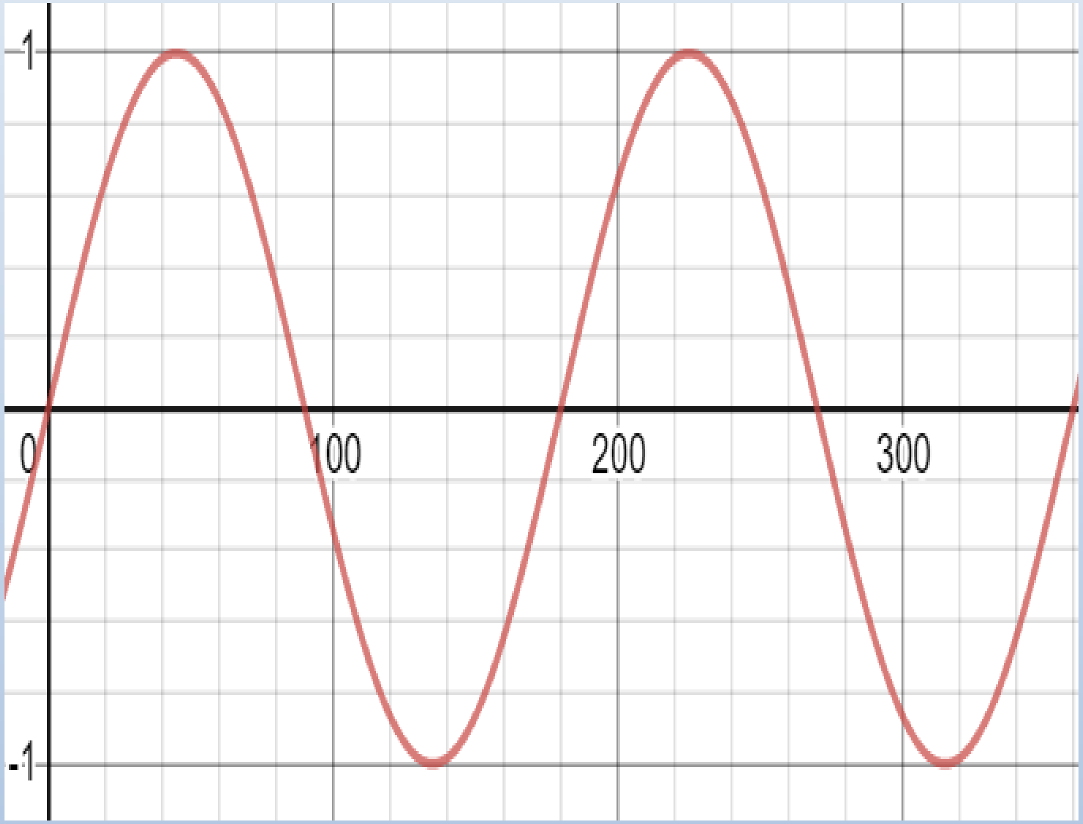
Example 3b

Sketch the graphs of the following from :

a)

b)

c)



Graph of stretched parallel to the -axis with scale factor 0.5. i.e. all the -coordinates are halved

3.1 Sine, cosine and tangent

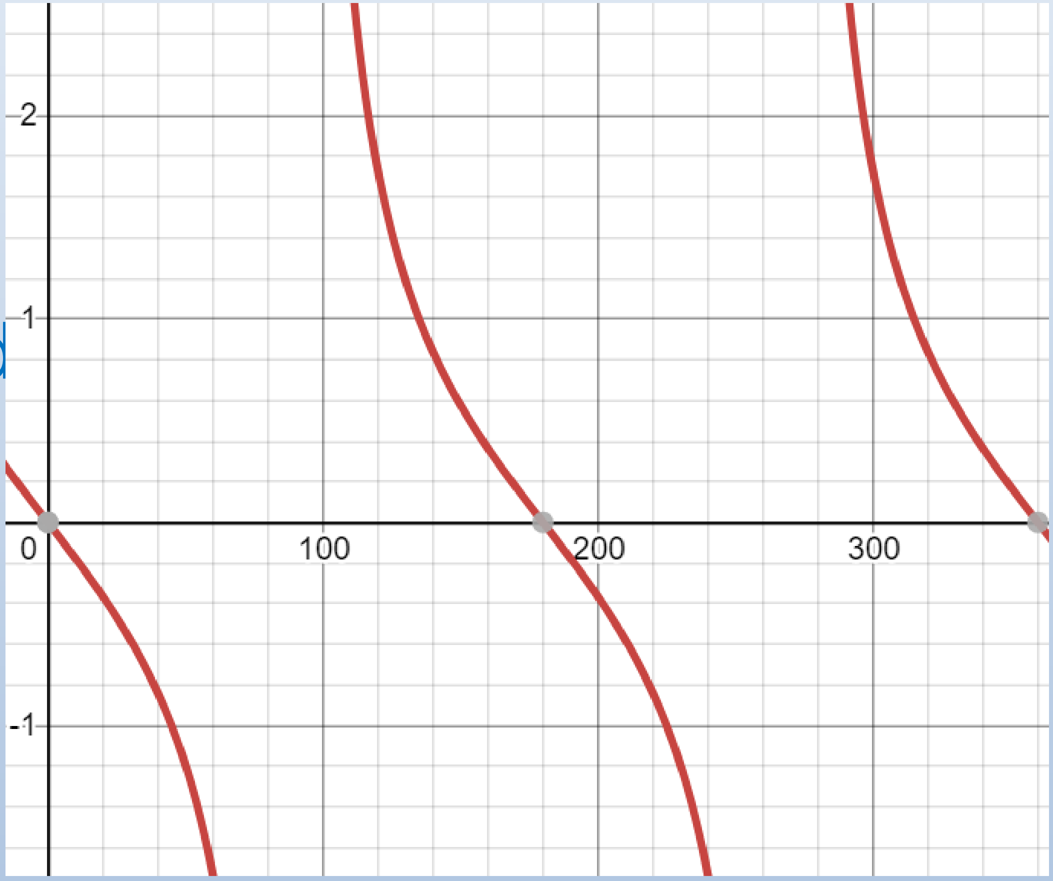
Example 3c

Sketch the graphs of the following from :

a)

b)

c)



Graph of reflected in the -axis